Evidence for a Possible Proton-Antiproton Bound State from Lattice QCD

Mushtaq Loan

Department of Physics, Zhongshan University, Guangzhou 510275, China.

School of Physics, The University of New South Wales, Sydney, NSW 2052, Australia

(Dated: May 11, 2006)

We have used standard techniques of lattice quantum chromodynamics to look for evidence of the spin-zero six quark flavour singlet state ($J^{PC} = 0^{-+}$) observed by BES Collaboration, and to determine the splitting between the mass of the possible proton-antiproton and the mass of two protons, its threshold. Ignoring quark loops and quark annihilation, we find indications that for sufficiently light quarks proton- antiproton is below the $2m_p$ threshold, making it a possible six-quark bound state.

PACS numbers: 11.15.Ha, 12.38.Gc,11.15.Me

Recently, the BES Collaboration in Beijing observed a near threshold enhancement in the proton-antiproton ($p\bar{p}$) mass spectrum from $J/\psi \rightarrow \gamma p\bar{p}$ radiative decay [1]. Fitting the enhancement with an S- wave Breit-Wigner resonance function, results in peak mass at $M = 1859^{+3}_{10}(\text{stat})^{+5}_{25}(\text{sys})$ with a total width $\Gamma < 30$ MeV/c^2. With a P-wave fit, the peak mass is very close to the threshold, $M = 1876.4 \pm 0.9$ MeV and the total width is very narrow, $\Gamma = 4.6 \pm 1.8$ MeV. This discovery is subsequently confirmed by the Belle Collaboration in different reactions of the decays $B^+ \rightarrow K^+ p\bar{p}$ [2] and $\bar{B}^0 \rightarrow D^0 p\bar{p}$ [3], showing enhancements in the $p\bar{p}$ invariant mass distribution near $2m_p$. Such a mass and width does not match that of any known particle [4].

Theoretical existence of possible proton-antiproton bound state has long been speculated in quark model and conventional nucleon potentials [5, 6, 7, 8]. However, it was only in a very recent study, made by Yan *et al* using the Skyrme model [9], that predicted mass and the width close with the experiment. Experimentally, the quantum numbers of $p\bar{p}(1857)$ are not well determined yet. The photon polar angle distribution was found consistent with $1+\cos^2\theta_\gamma$ suggesting the angular momentum is very likely to be $J = 0$. Making full use of general symmetry requirement and available experimental information the corresponding spin and parity are $J^{PC} = 0^{-+}$ [10]. In this letter we report first quenched lattice QCD calculation capable of studying the $p\bar{p}(1859)$.

In contrast with conventional baryons and mesons, it is difficult to deal with $q^m\bar{q}^n$ ($m+n \geq 3$) states in lattice QCD. For example, a $q^3\bar{q}^3$ state can be decomposed into couple of colour singlets states even in the absence of unquenched effect. The two-point function, in general, couples not only to the single hadron but also to the two-hadron states. If one allows such transitions, then - much like pentaquarks - it is not easy to separate and analyze the spectrum. In this study, we shall restrict ourselves to a 6-quark exotic with no transitions to regular mesons. We seek an operator that has a little overlap with the hadronic two-body states in order to identify the signal of our hexaquark state in lattice QCD. For this purpose, we construct our interpolating local operator based on the description of diquarks.

In Jaffe model [11, 12], each pair of $[ud]$ form a diquark which transforms like a spin singlet (1_s), colour anti-triplet ($\bar{3}_c$), and the flavor anti-triplet ($\bar{3}_f$). Therefore, for a diquark operator, one has [13]

$$Q^{i,a}_\Gamma(x) = \frac{1}{2}\epsilon_{ijk}\epsilon_{abc}q^T_{j,b}(x)C\Gamma q_{k,c}(x), \qquad (1)$$

where ϵ_{abc} is completely antisymmetric tensor, and (a, b, c) and (i, j, k) denote the colour and flavour indices, respectively. The superscript T denotes the transpose of the Dirac spinor and C is the charge conjugation matrix. The Dirac structure of the operator is represented by the matrices Γ, satisfying $\Gamma_{\alpha\beta} = -\Gamma_{\beta\alpha}$ (α and β , are Dirac indices) such that the diquark operator transforms like a scalar or pseudoscalar. The colour and flavour antisymmetries restrict the possible Γ's within 1, γ_5 and $\gamma_5\gamma_\mu$. Then the hexaquark hadron $p\bar{p}([ud][\bar{u}\bar{d}][\bar{u}u])$ emerges as a member with $S = 0$ and $I = 0$ in $(Q^{\bar{3}} \otimes \bar{Q}^3) \otimes Q^6 = ([\bar{15}_{cs}] \otimes [15_{cs}]) \otimes [21_{cs}]$ in SU(6) colour-spin representation and a flavour singlet in $(\bar{3}_f \otimes 3_f) \otimes 6_f$. With this picture the local interpolating operator for $p\bar{p}$ is obtained as

$$O_{p\bar{p}}(x) = \epsilon_{abc}Q^{i,a}_\Gamma(x)\bar{Q}^{i,b}_{\Gamma'}(x)Q'^{s,c}_{\Gamma''}(x) \qquad (2)$$

where $Q' = u^TC^{-1}\gamma_5\Gamma'' u$. This identification looks familiar if we represent one of the quarks by charge conjugate field: $q_aq_b \rightarrow \bar{q}_{Ca}q_b$, where $\bar{q}_C = -iq^T\sigma^2\gamma_5$. Then the classification of diquark bilinears is analogous to that of $q\bar{q}$ bilinears. We choose $\Gamma = 1$ and $\Gamma' = \Gamma'' = \gamma_5$. There are many more possibilities of constructing the operator even in the $I = 0$ channel. In principle testing various other interpolating operators for the best overlap with $p\bar{p}$ state should provide information on the wave function of the particle. In this study, however, we do not intend to pursue this issue any further. It is worth mentioning that with our proposed operator the description does not rely on highly correlated diquarks and it is straightforward to work out the hadron propagators in terms of quark propagators.

To examine the $p\bar{p}$ in lattice QCD, we generate quenched configurations a $16^3 \times 48$ lattice with tadpole-improved anisotropic gluon action [14] at $\beta = 2.4 - 3.2$, which correspond to lattice spacings of $0.48 - 0.37$ fm. Guage configurations are generated by a 5-hit pseudo heat bath update supplemented by four over-relaxation steps. These configurations are then fixed to the Coulomb gauge at every 500 sweeps. After discarding the initial sweeps, a total of 300 configurations for each β are accumulated for measurements . Using the tadpole-improved clover quark action on the anisotropic lattice [15] we compute the light-quark propagators at six values of the hopping parameter κ_t for bare quark mass in the range 10 - 100 MeV. We adopt a bin size of 30 configurations.

Hadron masses are obtained from the correlation functions of multiquark operators having the same quantum numbers as the hadrons in question. To obtain a better overlap with the ground state we used iterative smearing of gauge links and the application of the fuzzing technique for the fermion fields [16]. The application of fuzzing for two of the six quarks inside the $p\bar{p}$ flattens the curvature of the effective mass. The largest plateau in the region with small errors is obtained with fuzzed u- and d- quarks. We used this variant to calculate our correlation functions.

From the correlation functions we extract the mass (energy) by standard χ^2 fitting with multi-hyperbolic consine ansatz

$$C(t) = \sum_{i=1}^{n} A_i \cosh(tm_i) \tag{3}$$

The purpose of using the multi-state fit is to reduce the contamination from excited states. The fitting range $[t_{min}, t_{max}]$ for the final analysis is determined by fixing t_{max} and finding a range of t_{min} where the ground state mass is stable against t_{min}. We choose one "best fit" which is insensitive to the fit range, has high confidence level and reasonable statistical errors. Typical example of the effective mass plot is shown in Fig. 1. An impressive plateau with reasonable statistical errors is seem to terminate at $t = 40$. The 2-cosh fit for $p\bar{p}$ state gives the ground state mass consistent with that from 1-cosh fit. Statistical errors of masses are estimated by a single elimination jackknife method. We kept statistical errors under control by ensuring that analyzed configuration are uncorrelated, which is made possible by separating them by as many as 500 sweeps. The statistical uncertainties on our hadron masses are typically on the few percent level. In addition to the $p\bar{p}$ state we calculated the masses of the non-strange mesons π, ρ as well as the nucleon. Theses particle masses were used for scale setting and analyzing the stability of $p\bar{p}$ state, respectively.

Fig. 2 collects and displays the resulting particle masses extrapolated to the physical quark mass value

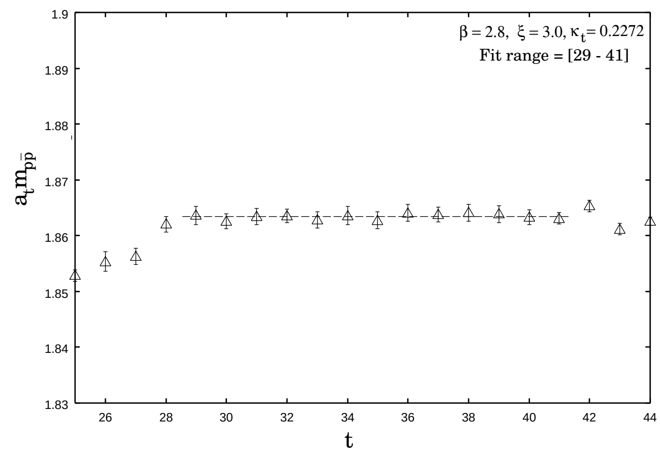

FIG. 1: Effective mass of $p\bar{p}$ state as a function of t_{min}. The dashed line represents fitted mass and its statistical errors.

using linear and quadratic fits. The difference between the two extrapolations gives some information about systematic uncertainties in the extrapolated quantities. Although, our quark masses are quite small and we have only five different quark masses at each β, both linear and quadratic fits essentially gave the identical results. We believe that the uncertainties due to chiral logarithms in the physical limit are significantly less dominant at our present statistics. However, since quenched spectroscopy is quite reliable for mass ratio of stable particles, it is physically even more motivated to extrapolate mass ratio instead of mass. Fig. 3 shows the chiral extrapolation of the $p\bar{p}$ to nucleon mass ratio at two different lattice sizes. It can be seen that quark mass dependence of this ratio seems to be weaker than individual hadrons (Fig. 2). Although our spatial lattice size is big enough for treating our measurements without finite-size effects, a finite-size consistency check was done on a $12^3 \times 36$ lattice at $\beta = 3.0$. With our statistical errors of order a few percent we did not find the size dependence in the hadron masses and mass ratios. The finite-size uncertainty in our quenched analysis turned out to be less than 0.2% for the $p\bar{p}$ ground state and less than 0.1% for the nucleon.

The major source of discrepancy among the lattice spacings from different observables is the quenching effect. The obtained ρ and N masses are compared to the experimental values and show a deviation less than $4-6\%$ for the lattice sizes explored here. Such a variance can be considered as usual quenching effect. Of course, simulations with dynamical fermions might be useful to eliminate this error all together. Inspired by the good agreement of the ρ and N masses with their experimental values, the scale was set by the ρ mass. We included a modest estimate of 5% quenching uncertainty in our analysis.

Finally, we performed a continuum extrapolation for the chirally extrapolated quantities in Fig. 4. The pos-

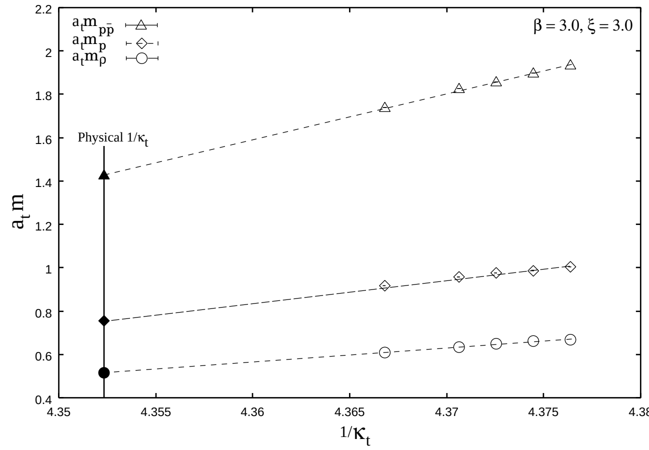

FIG. 2: Chiral extrapolation of hadron masses on one of our finest lattices.

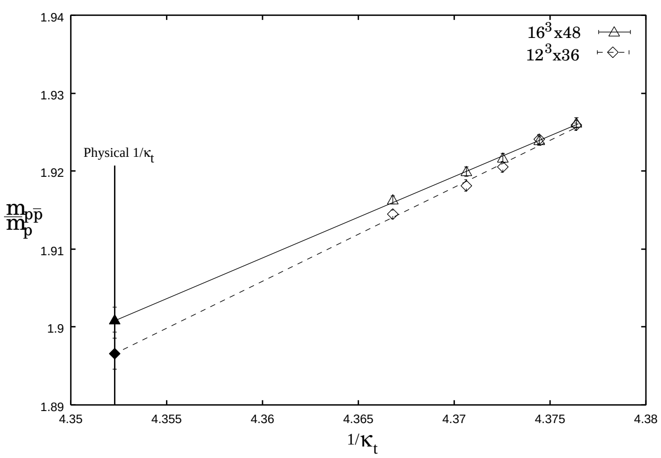

FIG. 3: Chiral extrapolation of the mass ratio on two different lattice sizes at $\beta = 3.0$.

sible error that might effect the simulation results comes from the scaling violation for our actions. Expecting that dominant part of scaling violation errors is largely eliminated by tadpole improvement, we extrapolate the results at finite a_s to the continuum limit $a_s \to 0$. Here we adopt an a_s^2- linear extrapolation for the continuum limit, because the leading order scaling violation for our fermion action is $O(a_s^2 \Lambda_{\rm QCD} m_q)$. We also perform an a_s-linear extrapolation to estimate systematic errors. In practice we use results at four finest lattice spacings, i.e., $\beta = 2.6 - 3.2$ for the continuum extrapolation, excluding results at $\beta = 2.4$, which appear to have larger discretization errors as expected from the naive order estimate. Performing such extrapolations, we adopt the choice which shows the smoothest scaling bahaviour for the final value, and use others to estimate the systematic errors. As can be seen from Fig. 4, the mass ratio again shows a weak dependence on the lattice spacing and varies only slightly over the fitting range. The four non-zero lattice spacing values of the ratio are within 0.04 - 0.06 standard deviations of the extrapolated zero lattice spacing result. This will make for unambiguous and accurate continuum extrapolation. Given the fact that the ratio does not show any scaling violations, we could also quote the value of this quantity on our finest lattice, which has the smallest error. Nevertheless, order 7% errors on the finally quoted values are mostly due to the chiral and the continuum extrapolation. The continuum results obtained are summarized in Table I. Using

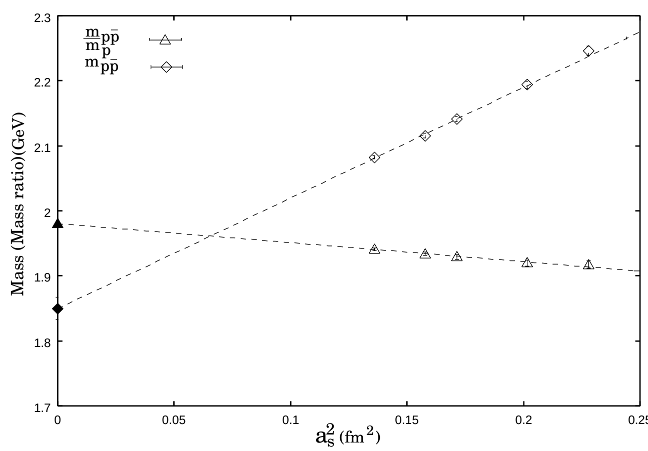

FIG. 4: Continuum extrapolation of the $p\bar{p}$ mass and the mass ratio $m_{p\bar{p}}/m_p$. The dashed lines are linear fits to the data in the range $0.136 \leq a_s^2 \leq 0.2015$. Solid symbols represent the predicted continuum values.

the physical nucleon mass $m_p = 938$ MeV, we obtain a continuum mass estimate of 1859±16 MeV for the $p\bar{p}$. Although the extrapolation of $m_{p\bar{p}}$ to the continuum limit shows significant discretization errors, the results from two extrapolation seems to be in good agreement, within errors. The lowest mass that we find in $J^{PC} = 0^{-+}$ channel is in complete agreement with the experimental value of $p\bar{p}$ mass [1]. The known isosinglet $X(1860)$ is obvious candidate to identify with the $p\bar{p}$ bound state we seem to have found on the lattice.

TABLE I: Chirally extrapolated results at finite lattice spacings. The continuum limit predictions were obtained by extrapolating the data for hadron masses as well as the mass ratio.

a_s^2 (fm^2)	m_p (Gev)	$m_{p\bar{p}}$ (GeV)	$m_{p\bar{p}}/m_p$	$m_{p\bar{p}} - 2m_p$ (GeV)	$-\delta E$ (GeV)
0.2279	1.191(5)	2.277(7)	1.917(6)	-0.106(4)	0.075(4)
0.2015	1.143(5)	2.193(6)	1.919(4)	-0.093(4)	0.069(3)
0.1715	1.113(3)	2.140(4)	1.929(4)	-0.086(3)	0.064(3)
0.1578	1.098(3)	2.115(3)	1.934(3)	-0.082(2)	0.058(2)
0.1360	1.080(2)	2.085(3)	1.941(2)	-0.074(2)	0.053(1)
$\to 0$	0.947(13)	1.851(24)	1.982(17)	-0.018(2)	0.016(2)

Now all prerequisites are available to measure the energy shift of $p\bar{p}$ state relative to the $2m_p$ threshold. To eliminate some of the statistical uncertainties we analyzed directly the ratio of correlators of the $p\bar{p}$ and the nucleon. This ratio is expected to take the single exponential form only at large t after contributions from excited states have died away. Following the usual procedure of looking for a plateau, we measured the energy shift δE for several different lattice spacings. It is clear from Fig. 5 that δE is varying rather slowly in the lattice spacing and we expect that this will lead to a bound state in the infinite volume limit. Note, that our continuum extrapolated $m_{p\bar{p}} - 2m_p$ is merely an illustration. Adopting an a_s^2-linear extrapolation we obtain a continuum result which implies that the energy shift of $p\bar{p}$ state does indeed move into the continuum with an attractive interaction between p and $\bar{p}$. This is a signature of a possible bound state, since we do not see the expected volume dependence in our simulations. However, the confirmation of such a signal will require a detailed study on very large volumes with increasing quark masses.

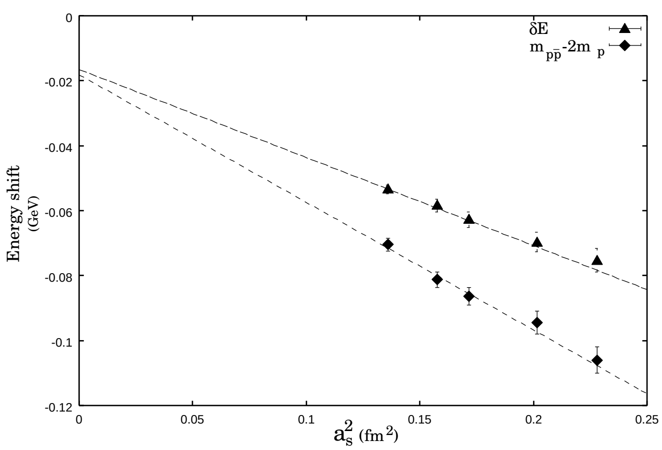

FIG. 5: Continuum extrapolation of the energy shift.

We have presented the results of the first lattice investigation on the $p\bar{p}$ state employing improved gauge and fermion anisotropic actions, relatively light quark masses as well as smearing techniques to enhance the overlap with the ground state of the particle. Our analysis takes into account all possible uncertainties, such as statistical, finite-size, and quenching errors when performing the chiral and continuum extrapolations. On the basis of our lattice calculation we speculate that the state is to be identified as a bound state of six quarks. However, a thorough examination of this question would require the implementation of flavour SU(3) violation. The $I = 0$, $p\bar{p}$ state couples to $4\pi\eta$ [10] through the $s\bar{s}$ component of the η in the quenched approximation. By giving the strange quark a larger mass would alter threshold which in turn would affect the manifestation of the bound state. Lattice calculations with varying quark mass are needed to confirm our results. Although it seems natural to expect that for sufficiently heavy quarks a bound state will remain, but only full, unquenched lattice calculations can confirm this. We did not make a systematic study of possible interpolating operators that are likely to have good overlap with $p\bar{p}$. This study would be also important. We plan to further develop this calculation to involve more interpolating operators.

I would like to thank R. Jaffe, C. Michael, C. Detar, S. Sasaki, C. Hamer and D. Leinweber for valuable suggestions. It is a pleasure to acknowledge Shan Jin for his hospitality at CCAST and stimulating discussions. We are grateful for the access to the computing facility at the Shenzhen University on 64 nodes of Deepsuper-21C. The computations were done using a modified version of the publicly available MILC code (see www.physics.indiana.edu/sg/milc.html). This work was supported by the Guangdong Provincial Ministry of Education.

[1] J.Z. Bai *et al.*, Phys. Rev. Lett. 91 022001 (2003)
[2] K. Abe *et al.*, Phys. Rev. Lett. 88 181803 (2002)
[3] K. Abe *et al.*, Phys. Rev. Lett. 89 151802 (2002)
[4] Particle Data Group, Phys. Rev. D **66**, 010001 (2002)
[5] E. Klempt, F. Bradamante, A. Martin and J.-M. Richard, Phys. Rep. **368**, 119 (2002)
[6] J.-M. Richard, Nucl. Phys. B (Proc. Suppl.) **86**, 361 (2000)
[7] A. Datta and O'Donnell, Phys. Lett. B **567**, 273 (2003)
[8] J.L. Rosner, Phys. Rev. D **68**, 014004 (2003)
[9] Mu-Lin Yan, Si Li, Bin Wu and Bo-Qiang Ma, Phys. Rev. D **72**, 034027 (2005)
[10] Chong-Shou Gao and Shi-Lin Zhu, Commun.Theor.Phys. **42**, 844 (2004)
[11] R.L. Jaffe, Phys. Rev. D **15**, 281 (1977)
[12] R.L. Jaffe, Phys. Rep. **407**, 1 (2005); Nucl. Phys. (Prog. Suppl) **142**, 343 (2005)
[13] S. Sasaki, Phys. Rev. Lett. **93**, 152001 (2004)
[14] C.J. Morningstar and M.J. Peardon, Phys. Rev. D **60**, 034509 (1999)
[15] M. Okamoto, *et al.*, [CP-PACS Collaboration], Phys. Rev. D **65**, 094508 (2002)
[16] P. Lacock *et al.*, [UKQCD Collaboration], Phys. Rev. D **51**, 6403 (1995)